

# CHAOS AND TURBULENCE

Ray Brown

EEASI Corporation  
Houston, Texas 77057

## Abstract

This paper will examine the relationship chaos and turbulence.

**Keywords.** Chaos, natural science, complexity, dynamical synthesis.

**AMS (MOS) subject classification:** 37D45.

## 1 Introduction

*A challenge to relating chaos and turbulence is that local turbulent dynamics are transient whereas chaotic dynamics can be easily examined in a static figure and analytically*

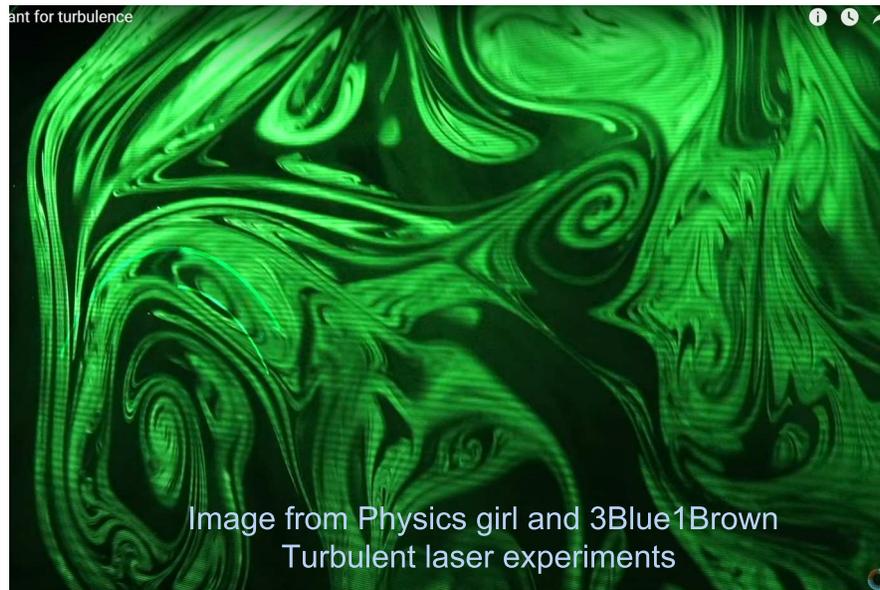


Figure 1: Physics experiment illustrating turbulent flow in a room by YouTube researchers Physics Girl and 3Blue1Brown

The challenge in linking the dynamics of chaos to the dynamics of turbulence is, in the case of the study of chaos, hyperbolic fixed points can be identified and the unstable manifold can be graphically produced. Figure 2 is an example of two unstable manifolds produced by direct analysis. In the case of turbulence, the *strange attractors* consist in a series of points that *vanish* before they can be computed.

The turbulent *strange attractor* is temporarily visible with the aid of a laser light on a smoke cloud and immediately dissipates under the *strong mixing* force of the turbulent dynamics. However, using technical devices, the swirls characteristic of unstable manifolds can be momentarily observed in the visible flow of the fluid under observation.

In order to simulate the *Flash Vortices*, the vortex of an *Ideal Fluid* is used [1], the example on page 30. The streamlines are concentric circles with a *Vortex Function*,  $\Psi(r)$ , where  $r$  is the radius from the origin. To simulate the distortion of the vortices seen in Fig. 1 from larger vortices as it moves within a fluid, it is composed with a linear rotation. Several examples follow.

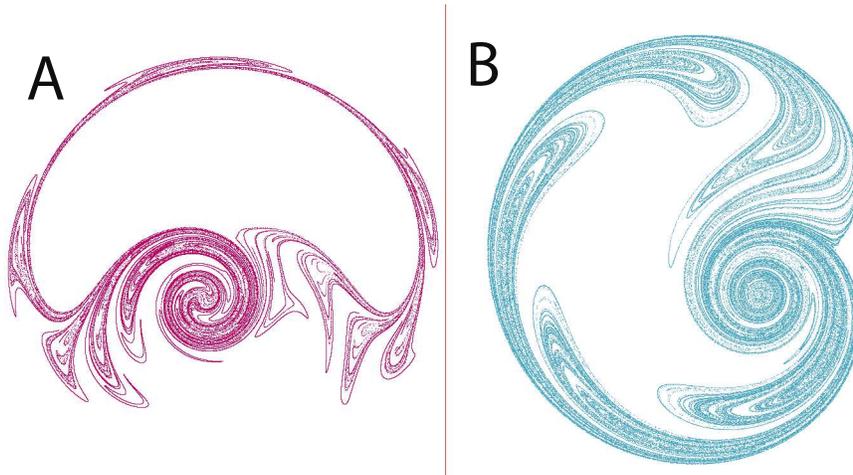


Figure 2: Two examples of unstable manifolds produced by analytically deriving the hyperbolic fixed points and the slopes of the associated unstable manifolds.

Figure 3, Plate A captures vortices evolving with fragments similar to segments of an unstable manifold. In a fluid, each of these swirls would vanish as fast as they formed and visible in the laser illuminated cloud.

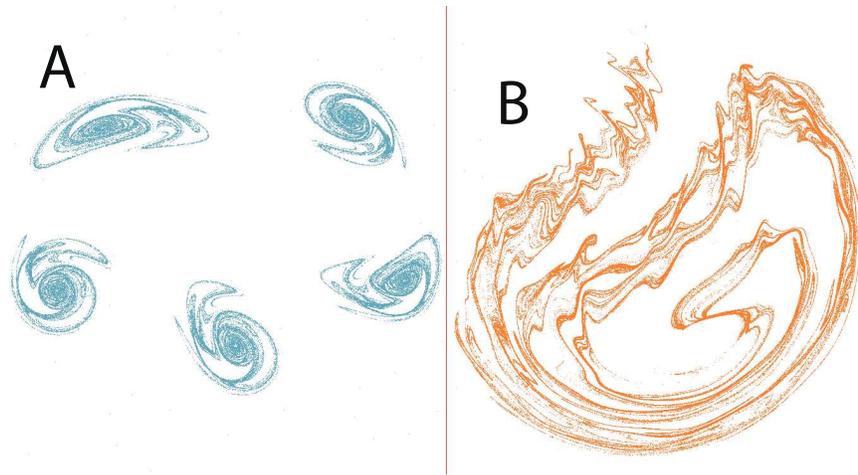


Figure 3: Plate A is a moving unstable manifold segment that momentarily forms in a turbulent flow and dissipates; Plate B is an unstable manifold perturbed by local dynamics

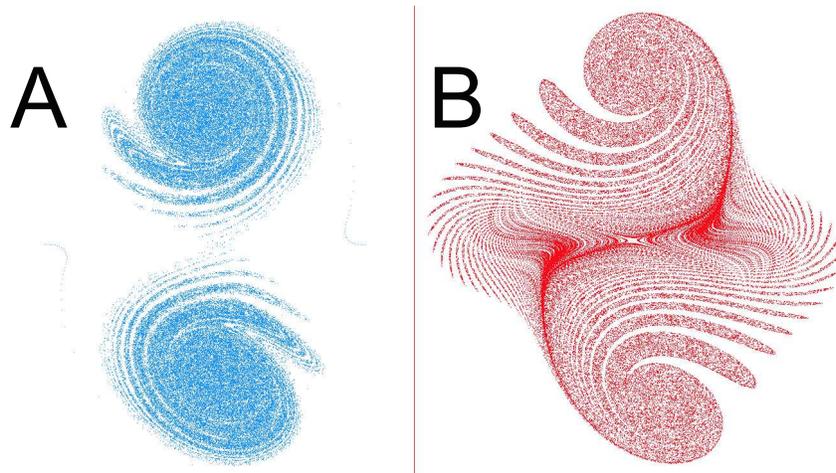


Figure 4: Two examples of evolving vortices: Plate A is an image with the vortices separated; In Plate B they have merged.

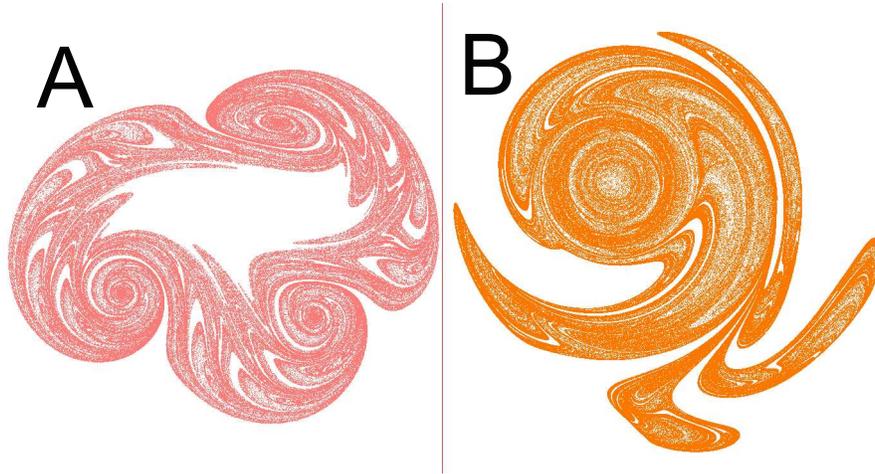


Figure 5: In these two plates are presented two vortices in separate parts of the flow. Both Plates supports the conjecture that turbulent flows create and destroy eddies by the interaction of vortices of varying size having different vortex functions  $\Psi(r)$

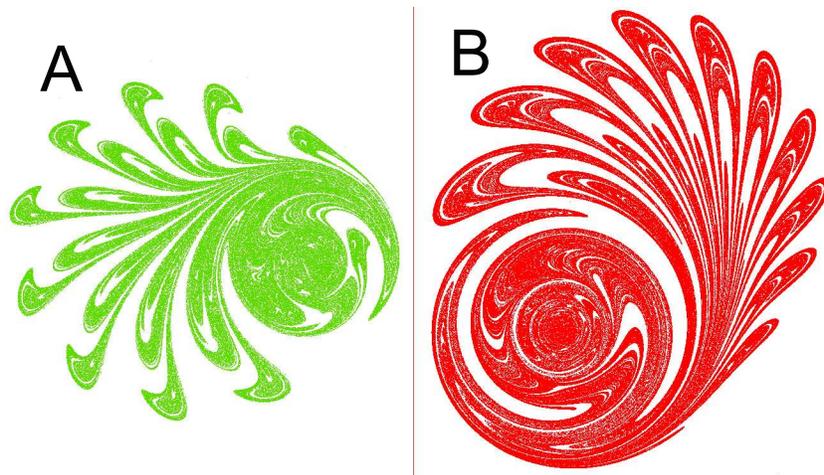


Figure 6: These two Plates are further support of the conjecture that the eddies and vortex functions are constantly evolving as the flow progresses. The lower the viscosity, the more variation in the vortex function

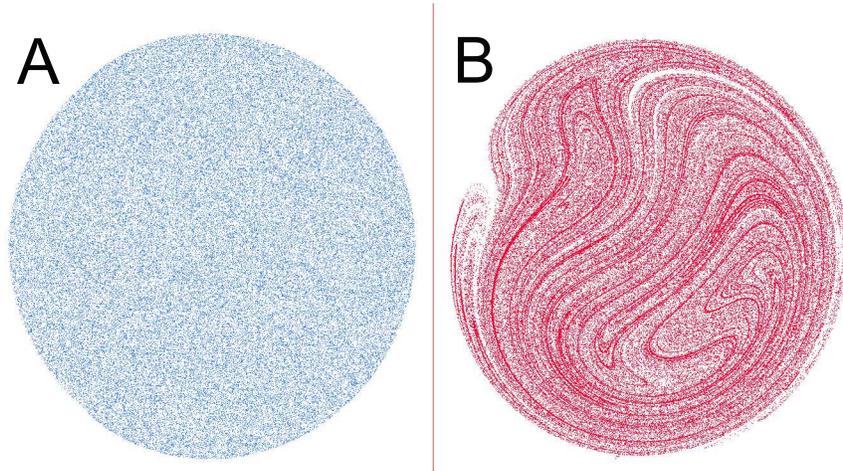


Figure 7: In plate A it is conjectured as the viscosity goes to 0, the eddy approaches a pure random process. Plate B is the eddy shortly before becoming a purely random process

## References

- [1] Chorin, A. J. and Marsden, J.E. [1990] *A Mathematical Introduction to Fluid Mechanics* Third Edition, Springer, New York.
- [2] Brown, R. [2020] "The Unilateral Shift IDE", DCDIS-B [2020] v2 no. 4
- [3] Brown, R. [2019] , *An Introduction to Infinitesimal Diffeomorphism Equations*, DCDIS-B V26s.